

Empirical Formulation of Stopping Distance for Loss Computation of CPW

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Abstract—A closed-form model for experiment based stopping distance is developed to calculate accurately conductor loss of CPW. The present improved Holloway and Kuester model has average accuracy of 5.62% against the experimental results from different sources in the frequency range 1 GHz – 60 GHz with conductor thickness 0.25 μ m - 1.58 μ m. The original Holloway and Kuester model has average accuracy of 13.7% and model of Ponchak *et al.* has 17.1 % against same set of experimental results. These models have been designed for use in (M)MIC's-CAD programs.

1. INTRODUCTION

Coplanar waveguide (CPW) has received considerable attention due to several advantages offered over conventional microstrips especially for monolithic microwave integrated circuits (MMIC's) applications. The available commercial software tools are capable of determining the characteristic impedance and propagation constant accurately. Normally they do not provide realistic estimates of line loss. Various closed-form equations have been used to determine the conductor loss in the CPW. These models suffer from limitations in accuracy and/or complexity. The available EM softwares are useful for the analysis of such components. However, the EM softwares usually do not compute realistically the conductor loss of a CPW. Moreover, accurate closed-form conductor loss model is incorporated in the circuit simulator for the fast analysis of the CPW based circuits.

Holloway and Kuester (HK) [1, 2] have used the concept of *stopping distance* with the standard perturbation method to compute the conductor loss of a coplanar waveguide (CPW) shown in Fig.1. They have taken the case of infinitely wide ground conductor with $c \rightarrow \infty$. The method is applicable to the conductor thickness (t) in the range $0.06 \leq t/\delta_s \leq 32$ where, δ_s is the skin- depth. The quasi closed- form HK model uses numerically generated stopping distance that is available in the tabular form; therefore, it is not convenient for the CAD application. Ponchak *et al.* [6] have shown that the HK model has an average error of 11% compared to the experimental results; whereas their own model has an 4.7% average error against their own experimental results. None of these models are tested against available extensive experimental results of

Haydl *et al.* [4, 5]. We have observed that both models have high average errors (17.07%, 13.69%) against the experimental results of Haydl *et al.*

The HK model along with the closed- form expressions of the stopping- distance is truly compact CAD oriented model for computation of conductor loss of CPW. However, theoretical stopping- distance is valid only for the isolated strip conductor [7]. It is not sufficiently accurate for computation of conductor loss of CPW. In the present work, we have explored the nature of stopping distance from the experimental results of Haydl *et al* [4, 5] and developed the experiment based CAD oriented expression of the stopping distance that significantly improves accuracy of the HK model.

2. NATURE OF STOPPING DISTANCE

The parameters of a coplanar transmission line are illustrated in Fig. 1, with ground-to-ground outer spacing $2c$, ground-to-ground inner spacing $2b = d$, slot width w , strip width $2a = s$, substrate thickness h and conductor thickness t .

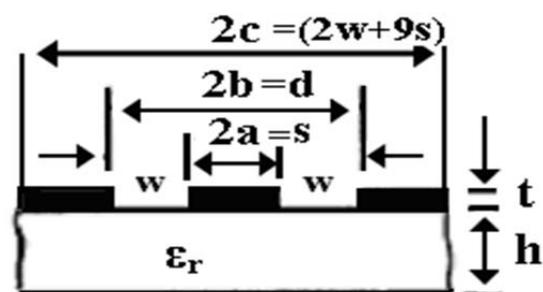


Fig. 1. Cross section of coplanar transmission line.

Holloway and Kuester [1,2] have generated a table for the normalized reciprocal stopping distance (t/Δ) for the isolated strip conductor of thickness t with 90° and 45° conductor edges, where, Δ is the stopping distance that avoids the edge singularity of the current density on the strip conductor while carrying out integration in the perturbation method. The

stopping distance is frequency dependent. Therefore, the normalized reciprocal stopping distance $y = (t/\Delta)$ is a function of the normalized skin-depth $(t/2\delta_s)$.

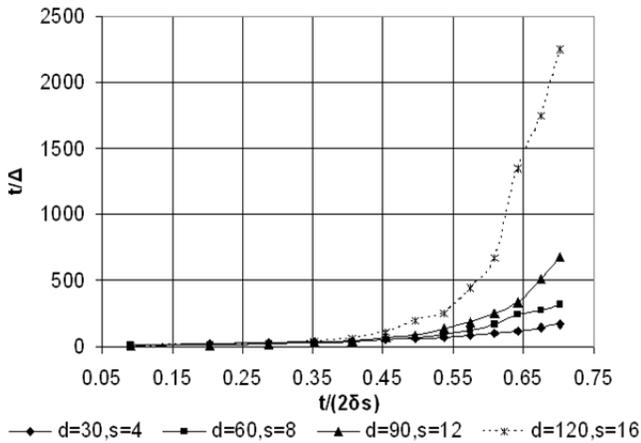


Fig.2. Structure dependence of stopping distance for $s/d = 0.13$, $t = 0.5 \mu\text{m}$, $\epsilon_r = 12.6$

In this paper, the nature of the stopping distance has been studied for different structural parameters of CPW. The experimental data demonstrate that the stopping distance Δ is structure dependent, which may comprise of conductor thickness, slot-width, strip-width, substrate thickness and ground-to ground inner spacing. Fig.2 shows dependence of the experimentally extracted (t/Δ) on the geometrical parameter of CPW on InP substrate. All the parameters are in μm . The (t/Δ) is both structure and frequency dependent. For the case $t/(2\delta_s) \leq 0.2$, (t/Δ) is structure independent while for $t/(2\delta_s) > 0.2$, i.e. at higher frequency, it is significantly structure dependent. For a fixed s/d , it is large for the wide central strip-width i.e. for a narrow slot-gap with thick strip conductor.

Fig. 3 compares the values of (t/Δ) at frequencies $f = 10, 40, 60 \text{ GHz}$ on InP substrate. The nature of the curve shows that with the increase in frequency, there is subsequent increase in (t/Δ) . Though at higher values of s/w ratio, a decrease in (t/Δ) has been observed.

Fig. 4 shows the comparison of the values of (t/Δ) for different s/d ratio at wide range of frequency on InP substrate. Till 25 GHz , a general trend of increase in value of (t/Δ) with increase in frequency has been observed for all s/d ratios. But after 25 GHz , the values of (t/Δ) at $s/d = 0.4$ exceeds those at $s/d = 0.13$ and change in nature of (t/Δ) has been observed. Thus at higher frequencies, there is sudden increase in (t/Δ) at $s/d = 0.4$ which results in a peak.

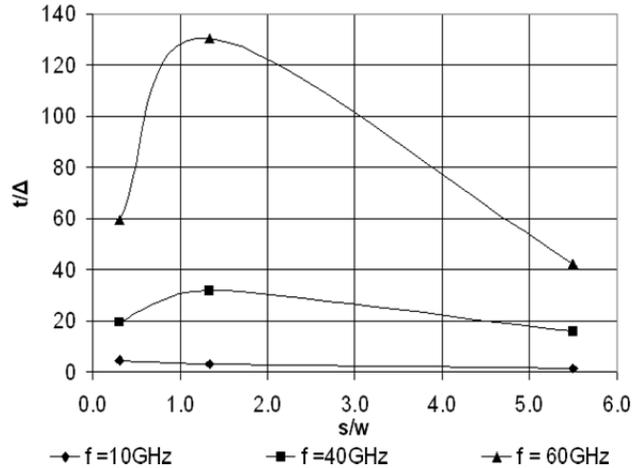


Fig.3. Variation of stopping distance for wide range of s/w ratio at $f = 10, 40$ and 60 GHz , $\epsilon_r = 12.6$, $d = 90 \mu\text{m}$, $t = 0.25 \mu\text{m}$

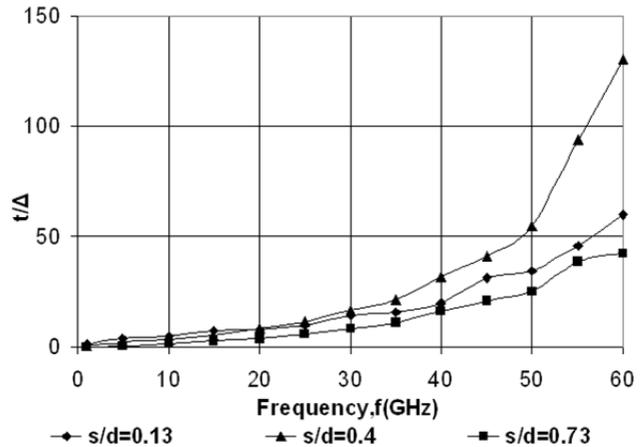


Fig.4. Structure dependence of stopping distance on frequency for different s/d ratio where $d = 90 \mu\text{m}$, $t = 0.25 \mu\text{m}$, $\epsilon_r = 12.6$.

3. CLOSED- FORM MODELS FOR STOPPING DISTANCE

The CPW structure of finite substrate thickness and finite width ground planes is shown in Fig.1. Holloway and Kuester have taken the CPW with infinite width ground conductors, $c \rightarrow \infty$. Following the perturbation method; as used by Holloway and Kuester, we obtain the following expression for the conductor loss of CPW shown in Fig.1, in terms of stopping distance,

$$\alpha \approx \frac{R_{sm} b^2}{16Z_0 K^2(k)(b^2 - a^2)} \left\{ \frac{1}{a} \ln \left[\left(\frac{2a}{\Delta} - 1 \right) \left(\frac{b-a+\Delta}{b+a+\Delta} \right) \left(\frac{c+a-\Delta}{c-a-\Delta} \right) \right] + \frac{1}{b} \ln \left[\left(\frac{2b}{\Delta} + 1 \right) \left(\frac{b-a+\Delta}{b+a-\Delta} \right) \left(\frac{c-b-\Delta}{c+b-\Delta} \right) \right] \right\} \quad (1)$$

where, $a = s/2$, $b = s/2+w$, stopping distance (Δ) is measured from edge of the central strip. The surface impedance R_{sm} of the strip conductor of thickness t is obtained from the expressions given in [10]. The characteristic impedance of the CPW shown in Fig.1 is obtained from the standard expression [8, 10]. Ponchak *et al.* [6] have taken $2c = (2w+9s)$. The elliptic integral ratio $K(k')/K(k)$ is evaluated by using the closed-form expressions [11].

We extracted the stopping distance (Δ) from the extensive experimental results of the conductor loss of CPW provided by Haydl *et al.* [4, 5] for the frequency range 1 GHz to 60 GHz. We noted that the theoretical reciprocal normalized stopping distance (t/Δ) obtained by Holloway and Kuester [1] follows the experimental results for the reciprocal normalized skin-depth ($t/2\delta_s$) ≤ 0.5 . For the thin strip conductors used in MMIC technology; there is a significant deviation in some frequency range. Therefore, we obtained the following curve- fitted expression for normalized stopping distance from the experimentally extracted stopping distance from one set of graphical results of Haydl *et al.*

The parameter used are- $s/d = 0.13$, $t = 0.25 \mu\text{m}$, $h = 500 \mu\text{m}$, $\epsilon_r = 12.6$ (InP substrate) $d = 30 \mu\text{m}$, $s = 4 \mu\text{m}$, $w = 13 \mu\text{m}$ over frequency range 1 GHz – 60 GHz. In general (t/Δ) is structure dependent. However, for simplicity we have obtained the following empirical expression that is only frequency dependent through the normalized conductor thickness parameter $t/(2\delta_s)$ over the range $0.045 < x < 2.0$:

$$y = \begin{cases} 1668x^4 - 1504.1x^3 + 595.85x^2 - 18.823x + 0.6874, & \text{for } 0.045 \leq x < 0.7 \\ 2394.4x^4 - 5861.9x^3 + 4839.1x^2 - 1025.3x - 51.873, & \text{for } 0.7 \leq x < 2.0 \end{cases} \quad (2)$$

4. COMPARISON TO EXPERIMENTAL AND OTHER RESULTS

The accuracy of the original Holloway- Kuester (HK) model [2], Ponchak, Matloubian, and Katehi (PMK) model [6] and present improved HK (IHK) model are compared against two sources of experimental data. The first source of 336 experimental data on the conductor loss of CPW is the graphical experimental results of Haydl *et al.* [4, 5] on GaAs ($\epsilon_r=12.9$) and InP ($\epsilon_r=12.6$) substrates of thickness 0.5 mm, conductor thickness $t=0.25, 0.5, 1.0 \mu\text{m}$ and $s/d = 0.13, 0.4, 0.73$ in the frequency range of 1 GHz – 60 GHz. The second source for the 36 experimental data points is the graphical experimental results of Ponchak *et al.* [6] for the CPW on GaAs, InP and Si substrates for characteristic impedances $35 \Omega, 50 \Omega, 65 \Omega$.

Fig.5 illustrates typical comparisons of models against the experimental results of Haydl *et al.* in the frequency range 1 GHz – 60 GHz for conductor loss of a CPW structure on InP substrate with ground-to-ground inner spacing, $d=60 \mu\text{m}$. For a fixed strip width (s) / slot-gap (w) ratio, the conductor loss decreases with increase in the strip-width and the slot-gap. The narrow and wide slots are more in error for IHK model; this observation is being supported by [2], which is based on the assumption that the edge of the strip is isolated from other strip edges. There is decrease in error with increase in conductor thickness, s/d ratio and frequency. The model HK, model PMK and model IHK have average error 13.69%, 17.07% and 5.75% respectively and maximum error 34.32%, 26.75% and 19.69% respectively.

Fig. 6 shows % average deviation results for CPW structure with $t=1.58\mu\text{m}$, $\sigma=4.1 \times 10^7 \text{ S/m}$, $f=23 \text{ GHz}$ with wide range of s/w for Si $\epsilon_r=11.9$, $h=360\mu\text{m}$. The model HK, PMK and IHK have average error 13.79%, 5.52% and 4.71% respectively and maximum error 34.98%, 24.6% and 32.73% respectively.

Ponchak’s [6] curve-fitted closed-form equation for attenuation, the model PMK, is substrate specific but the model IHK handles any substrate and any dimension. The model PMK had been only compared with experimental results in [6] but not with [4]-[5]. When compared, it has been observed that overall model PMK is in 17.07% error, for GaAs error is 8.3% and for InP error is 14.55%. In [6], it has been claimed that the model HK has 11% average error, which shows excellent agreement with the comparison made here. The 2% deviation might be due to the reading error. An average error of 25.09 % is obtained with the model IHK using Wheeler’s incremental inductance rule [9], which is 40% for the model PMK [6]. The model HK and model PMK have overall average error 13.7%, 15.62% respectively and overall maximum error 43.37%, 32.95% respectively. The average error for the model PMK had been 6% [6]. Overall, model IHK is preferable, which is showing excellent correlation with both the experimental results [4]-[6], and is giving average error of 3.73% and maximum error 26.79%.

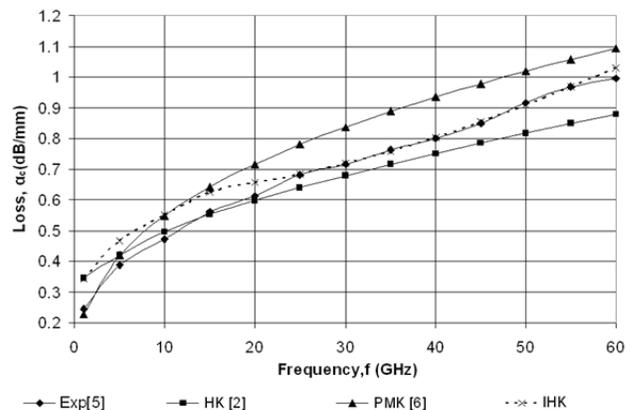


Fig. 5. Comparison of conductor loss models against experimental results [5] for $\epsilon_r=12.6$, $s/d=0.73$, $d=60\mu\text{m}$, $t=0.5\mu\text{m}$, $h=500\mu\text{m}$

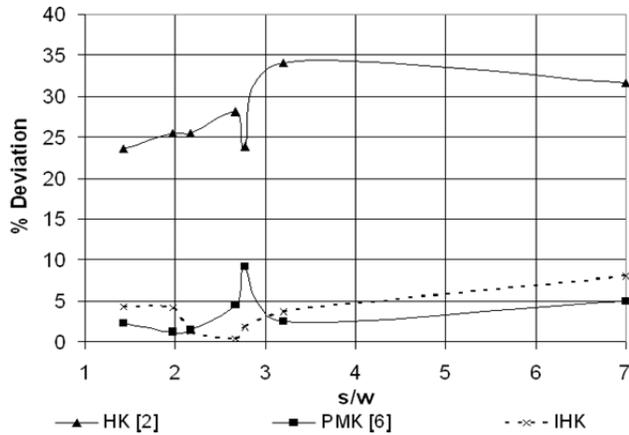


Fig.6. Comparison of conductor loss models against experimental results [6] for $\epsilon_r=11.9$, $t=1.58\mu\text{m}$, $h=360\mu\text{m}$, $\sigma=4.1 \times 10^7\text{S/m}$

Since model IHK has been derived from measured attenuation, it also accounts conductor, dielectric and radiation loss, but the conductor loss dominates for the GaAs and InP substrates [6]. Outcome of the comparison is summarized in table-1.

5. CONCLUSIONS

This paper has presented an experiment-based closed-form model for the stopping distance of CPW and presented the improved HK model to compute conductor loss of CPW. The validity of the model is tested in the frequency range 1 GHz – 60 GHz for the conductor thickness 0.25 μm -1.58 μm . The present IHK model has average error of 5.62% against the experimental results from two sources. The original Holloway- Kuester model has average error 13.7%. The nature of the stopping distance has also been studied and has found to be dependent on structural parameters and frequency.

Table-I: % Deviation of models against experimental results of Haydl [4, 5], Ponchak [6]

[Data range: $t = 0.25 \mu\text{m} - 1.58 \mu\text{m}$; Freq = 1 GHz – 60 GHz]

Models	Haydl		Ponchak	
	Av	Max	Av	Max
HK[2]	13.69	34.32	13.79	34.98
PMK[6]	17.07	26.75	5.52	24.6
IHK	5.75	19.69	4.71	33.26

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REFERENCES

- [1] C.L. Holloway, and E.F. Kuester, "Edge shape effects and quasi closed form expressions for the conductor loss of microstrip lines," *Radio Sci.*, vol. 29, No.3, 1995, pp. 539-559.
- [2] C.L. Holloway, and E.F. Kuester, "Quasi- closed form expression for conductor loss of CPW lines," *IEEE Trans. Microwave Theory Tech.*, vol. 43, No. 12, 1995, pp. 2695-2701.
- [3] C.L. Holloway, "Expressions for the conductor loss of strip-line and coplanar-strip (CPS) structures," *Microwave and Optical Tech. Letters*, vol. 25, No. 3, 2000, pp.162-168.
- [4] W.H. Haydl, "Experimentally Observed Frequency Variation of the Attenuation of Millimeter-Wave Coplanar Transmission Lines with Thin Metallization," *IEEE Microwave and Guided Wave Letters*, vol. 2, No.8, 1992, pp. 322-324.
- [5] W.H. Haydl, "Attenuation of Millimeter-wave Coplanar Lines on Gallium Arsenide and Indium Phosphide over the Range 1 - 60 GHz," *IEEE MTT-S Digest*, 1992, pp. 349-352.
- [6] G.E. Ponchak, M. Matloubian, and L.P.B. Katehi, "A Measurement-Bsed Design Equation for the Attenuation of MMIC-Compatible Coplanar Waveguides," *IEEE Trans. Microwave Theory Tech*, vol. 47, No. 2, 1999, pp. 241-243.
- [7] W. Heinrich, "Quasi-TEM description of MMIC coplanar lines including conductor-loss effects," *IEEE Trans. Microwave Theory Tech*, vol. MTT- 41, No. 1, Jan 1993, pp. 45-52.
- [8] K.C. Gupta, *Microstrip Lines and Slotlines*, 2nd Edition, Artech House, Norwood, MA.
- [9] A.K. Verma, Nasimuddin and H. Singh, "Conductor loss of the Coplanar Waveguide with conductor backing and top shield," *APMC*, 2004.
- [10] R.N. Simons, *Coplanar Waveguide Circuits, Components, and Systems*, A John Wiley & Sons, Inc., Publication, 2001, pp.203-217.
- [11] R.E. Collin, *Foundation of Microwave Engineering*, 2nd Edition, McGraw Hill, USA.
- [12] P. Majumdar, *Analytical Modeling of Single-Layer and Multilayer Planar Transmission Lines* (Published doctoral thesis), University of Delhi, New Delhi, India, 2012.